
EXPONENTIAL FUNCTIONS – AN INTRO

❑ INTRODUCTION

This chapter assumes that you have studied linear functions (straight lines) and quadratic functions (parabolas).

❑ THE RACE

Three functions are having an automobile race. Each of the three functions has a different type of engine, given by the following formulas:

$$2t \quad t^2 \quad 2^t \quad \text{[where } t \text{ is time]}$$

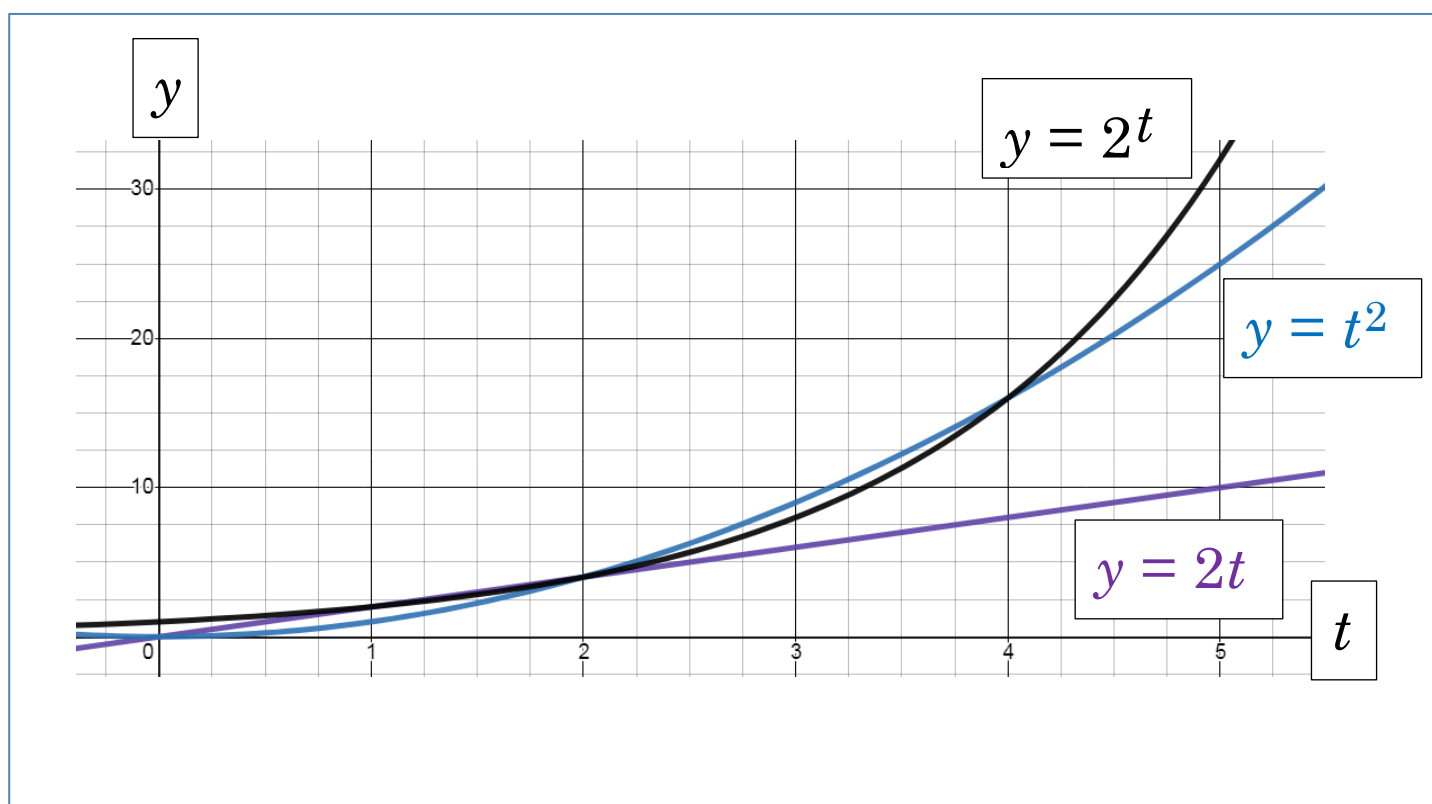
We're going to make a table of all three functions of time, and use it to figure out which one is winning the race at various moments in time, and which one will be the ultimate winner if we let *time* grow larger and larger (further into the future).

t	$2t$	t^2	2^t	
0	0	0	1	2^t is in the lead.
1	2	1	2	Now $2t$ and 2^t are tied.
2	4	4	4	At $t = 2$, there's a 3-way tie.
3	6	9	8	t^2 pulls ahead.
4	8	16	16	$2t$ is fading, while t^2 and 2^t are tied.
5	10	25	32	2^t now has the edge.

6	12	36	64	$2t$ and t^2 are fading away.
10	20	100	1,024	Now it's not even a race anymore.
20	40	400	1,048,576	This is getting ridiculous!

For all the t -values from 0 to 4, the function 2^t is sometimes ahead, sometimes tied, and sometimes behind. But, starting at $t = 5$, the function 2^t grows leaps and bounds beyond the other two functions. We say that the function 2^t is growing **EXPONENTIALLY**.

You may recall that $2t$ is a linear function, while t^2 is a quadratic function. We're now going to graph all three functions on the same grid to get a picture of what's going on.



Moreover, here's how powerful an exponential function can be even when compared to a “massive” quadratic function:

Let's compare the two functions

$$y_1 = 100x^2 \quad \text{and} \quad y_2 = 2^x \quad \text{[The 100 makes } y_1 \text{ quite large.]}$$

For $x = 5$: $y_1 = 2,500$ and $y_2 = 32$ [y_2 seems quite weak.]

For $x = 10$: $y_1 = 10,000$ and $y_2 = 1,024$ [y_2 is still way behind.]

For $x = 12$: $y_1 = 14,400$ and $y_2 = 4,096$ [Does y_2 even have a chance?]

For $x = 25$: $y_1 = 62,500$ and $y_2 = 33,554,432$ [Now y_2 can't be stopped!]

And for my next statement, you'll have to trust me:

*No matter what large number
you put in front of the x^2 ,
the 2^x will eventually overtake it
and remain in the lead forever,
with the gap between them
growing larger and larger.*

In fact, you can even change the exponent on the x^2 to a really large number, and 2^x will still eventually leap ahead. So, for example, in a race between x^{78} and 2^x , the 2^x will eventually win the race (although an unbelievably gigantic value of x would be needed for this to happen, perhaps even beyond the capability of your calculator.)